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STRUCTURE-BORNE SOUND POWER INPUT IN LOW-MOBILE CONCRETE PLATES

Tor Kihlman
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CONTENTS

PREFACE

This report deals with problems in the measurement, interpretation and use of point mobility on low mobile structures such as concrete plates. The extra difficulties in measurements on such objects are caused by the pronounced local deformation in the point of excitation which is superimposed upon the bending wave field. Earlier work at the department had also indicated that these local deformations might give rise to a noticeable local dissipation of energy. The measurement problems were found to be quite substantial and much time were spent on measurements that did not give very clear results. To find "the final solution" would have demanded many more measurements than could be included in this project. Therefore the report only partially answers the questions originally asked.

We are much indebted to Dr Björn Petersson for valuable discussions and to Mr Arne Jagenäs for spending much time on the measurements.

SUMMARY

This report deals with some aspects on point- and transfer mobility measurements on low-mobile structures, i.e. concrete plates. An important feature of such structures is that the local, springlike, deformation under the indenter is often big compared to the global deformations in e.g. bending. This circumstance is of importance for the measurement technique and for the interpretation of measurement results. The purpose of our work has been to develop and confirm results obtained by Petersson [1].

Two basic questions basically, are investigated:

1) In point mobility measurements on low mobile structures; which influence does the local stiffness in the surface of the plate have?

2) The power input in a point from the source of excitation can be calculated from measured values of the force and the velocity at the point, taking the real part of the product of the force and the velocity. This power input (P_{IN}) may be divided in two parts, one part that is locally absorbed in the local deformation process and one part that is fed into the structure as propagating waves (P_D). The reason for this is that such a local absorption of power underneath the Force Distribution Indenter (see (Fig. 1.4), implies that only a fraction of the power supplied by the source really propagates. This means that $P_{IN} > P_D$. The task in this investigation is to determine the order of magnitude of the local absorption of power. Some results obtained by Petersson, [1], have indicated that such local losses couldn't be neglected in measurements on thick concrete structures.

Various mobility measurements have been made to verify some models and assumptions about the properties of concrete plates with respect to the local deformation of the surface close to the indenter.

Difficulties are encountered because it is difficult to distinguish between the local deformation field (of reactive character), the direct radiating field and the reverberant field.

In order to obtain reliable mobility measurement results it is necessary to use an indenter of realistic size. Measurements with impedance heads are unreliable because their indenter area is too small and their force transducers

normally too soft. The force distribution indenter (FDI) proposed by Peterson is a better device for such measurements on concrete structures. With this FDI the point velocity is measured in the center of the indenter, see fig. 1.2. When possible it is still better to measure the velocity on the opposite side of the concrete structure provided that the location of the indenter can be determined with high accuracy.

Local inhomogeneities of the concrete have not been found to cause any noticeable spread in our data.

Some results

The experiments were performed with concrete slab floors as test objects. Anisotropy, inhomogeneities and the unprecise knowledge of the flexural properties caused no serious problems. However, the uncertainties of various methods for loss-factor measurement, and also the uncertainties about the flexural properties, made our comparisons of estimates somewhat difficult. These difficulties should be possible to overcome. The various measurements described all require advanced equipment, experienced staff and suitable software for FFT-analysis and post-processing.

The measurements on a concrete slab have been highly reproducible, the Force Distribution Indenter seems to even out the influence of any local inhomogenities. No fluctuations (except those attributed to modal behaviour) have been observed.

The measurement of the structure-borne sound power input by means of force- and mobility-measurements, is not heavily influenced by an absorption of power locally. The propagating power may be less than the measured power input, which should be considered as a bias error in the estimate. However, in our measurements the magnitude of any locally absorbed power has been found much less than the propagating fraction. Nor has any reasonable physical model been found that would indicate anything but negligible local losses. It can be concluded that measurements of the real part of the point mobility, using the FDI, can be used for calculations of structure-borne sound power input with sufficient accuracy.

The local deformation of the surface of the structure, which adds to the deformation caused by flexural waves, governs the point mobility of a low-mobile concrete slab almost completely. This local deformation, which is practically completely reactive, decays with increasing distance to the indenter. A "Boussinesq-type-" static description of that decay is suitable also at higher frequencies, as suggested by Petersson [1] amongst others. (This is treated in detail by Ljunggren [9]).

Further away from the indenter, (at about $k \cdot r = 3$ on our particular measurement object), the direct field gets lost in a diffuse-field, and the phase of the transfer mobility is growing rapidly with increasing frequency, because the phase of the reverberant velocity field is random.

INTRODUCTION

Earlier projects in the field of structure-borne sound have shown a need for investigations of some matters concerning measurement technique and the definition of point mobility in the context of thick concrete structures.

The point mobility on a concrete surface consists of one part related to the bending of the plate (global deformation), and one part that consists of the plate surface deformation under the loading point due to the elastic properties of the concrete. The former is independent of the size of the loading indenter, but the latter component decreases with increasing size of the indenter. These two velocity components added, is the velocity that goes into the definition of the point mobility. The point mobility is therefore dependent of the indenter size. The bending-wave solution for <u>thin</u> plates however, gives a point mobility that is independent of the area of the loading structure in contact with the concrete structure. The local deformation therefore arises difficulties in the definition of the point mobility of heavy concrete structures, and one project task is to perform measurements in order to determine the influence of the local deformation. The power input in a point from the source of excitation can be calculated from measured values of the force and the velocity at the point, taking the real part of the product of the force and the velocity. This power input (P_{IN}) may be divided in two parts, one part that is locally absorbed in the local deformation process and one part that is fed into the structure as propagating waves (P_D). This means that $P_{IN} > P_D$. The task in this investigation is to determine the order of magnitude of the local absorption of power.

In chapter 1, a model of the measurement objects, two concrete plates, and the different components of motion is established.

In chapter 2 a few approaches are discussed, to estimate the order of magnitude of a local absorption of power in the vicinity of the Force Distribution Indenter (FDI).

Chapter 3 deals with the spatial variations of the surface particle velocity close to and further away from the indenter. When the plate is low-mobile, the local deformation constitutes a spring which increases the velocity of the point. This local deformation decays rapidly with the distance to the indenter. For transfer mobility predictions, it is valuable to know this behaviour. It also stresses the importance of the selection of position of the velocity transducer.

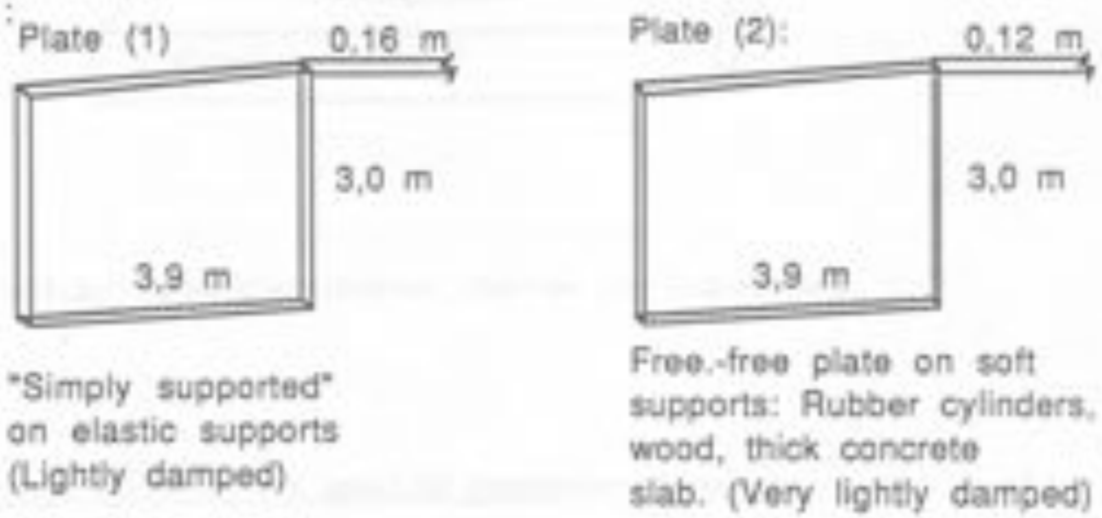
The direct, propagating field decays with distance from the point of excitation. On finite plates the boundary reflections give rise to a diffuse field that contributes to the total velocity. An evaluation of the transfer mobility measurements gives information about this relationship. Also, velocity measurements at various positions on the plate give an idea of the velocity distribution.

Chapter 4 contains experimental results and interpretations according to the questions above. The influence of local inhomogenities in the concrete on the results is studied.

1. DESCRIPTION OF MEASUREMENT OBJECTS AND THE MODEL OF COMPONENTS OF MOTION.

1.1. Description of the measurement objects

The measurements have been performed on two concrete slabs, of a size that is typical for floors and partition walls in multistore buildings in Sweden. The results should apply on most concrete plates. Plate (1) is "simply supported" on somewhat elastic supports, which gives a light damping along the edges due to transmisssion losses. Plate (2) is softly supported at two edges and is very lightly damped.



Figur 1.1. The measurement objects.

1.2. Description of measurements and devices

A GENRAD FFT-Analyser 2515 was used for all measurements, which allowed highly coherent measurements. Especially in the low-frequency domain, it is difficult to obtain a sufficient measurement accuracy. The transducers and charge amplifiers used were Brüel & Kjaer force transducer 8200, 2.3 g accelerometers 4344 and charge amplifiers 2635. One accelerometer was mounted inside a Force Distribution Indenter, described below.

The difficulty in measuring both the force and the velocity correctly on low-mobile structures, discussed by Petersson [1] and others [12] resulted in a measurement device, an annular indenter that distributes the force on the surface of the structure. The indenter was used in all measurements, because it enables velocity measurements in the center of the point with no extra masses or forces affecting the velocity transducer. It will be denoted "Force Distribution Indenter",for short "the FDI". See also Petersson [1].

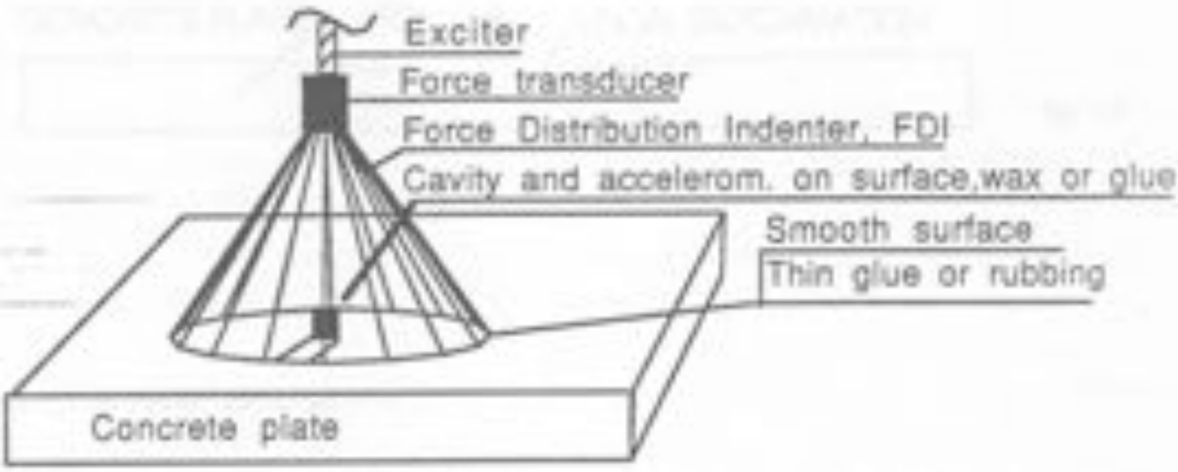


Figure 1.2. Force Distribution Indenter and Transducers.

1.3. Description of the model of components of motion.

The questions stated in the summary concern different approaches to point and transfer mobility measurements. Some concepts have to be defined in the context, to make the discussion comprehensive.

The mobility Y is the ratio between the velocity (v) and the force (F) that excites the structure:

$$Y = \frac{v}{F} \tag{1.1}$$

The velocity of the plate surface right under the Force Distribution Indenter can be considered to be the result of two types of deformation.

- the local deformation due to a stiffness in the elastic plate, independent from the overall flexural deformation of the plate.

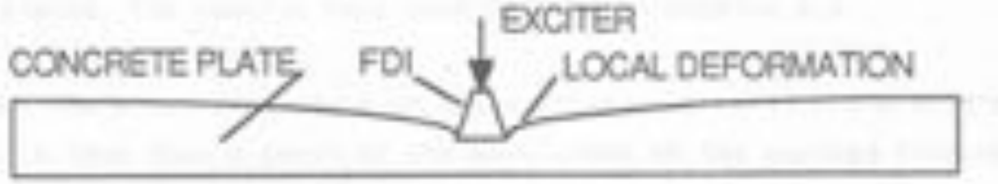


fig 1.3

Figure 1.3. The local deformation.

The deformation is proportional to the force (F) and the radius of the indenter (a) as an expression for the average deformation of the surface of an elastic half-space under a loading soft indenter shows [1]:

$$< d > = 0.54 \frac{1 - \nu^2}{Ea} \cdot F \qquad (2.2\ a)$$

Equation (2.13) and Fig.2.3 describe the deformation in detail.

- the overall global deformation due to the bending waves in the plate.

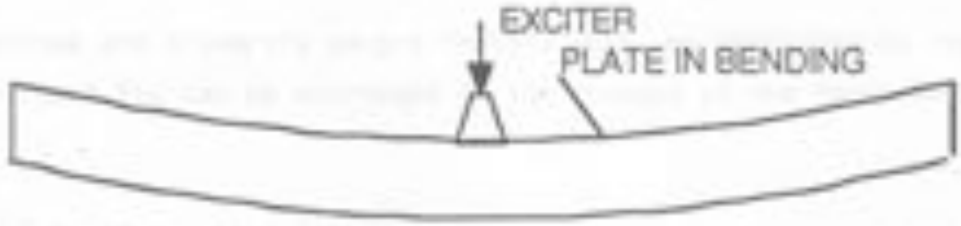


fig1.4

Figure 1.4. The deformation from bending waves.

We can assume in a simple theoretical model, that the local deformation is described by the theory for an elastic halfspace, and that the flexural deformation is described by the theory for bending waves in thin plates. These assumptions limit the applicability of the model. Ljunggren [9] has recently made a profound theoretical investigation of the local deformation in thick concrete plates. The results have been applied in section 2.4.

Assume that the cross dimensions of the excitation area (i.e the size of the indenter) is less than a tenth of the wavelength of the excited flexural wave, and the frequency of excitation is less than kh = 1. If shear deformations and rotational inertia are included in the solution, this results only in a near field with a spring-like character. The effect of these added considerations to the simplified bending wave equation, is negligible under the conditions mentioned. The excitation area may thus be regarded as a point in the context of excitation of bending waves, and the excitation area affects only the magnitude of the local deformation.

The point mobility (Y_0) is the ratio between the total velocity and the excitation force:

$$Y_0 = \frac{v_0}{F} \qquad (1.2)$$

v_0 consists of two different parts that are superimposed: v_{LOC} due to the local deformation and v_D due to bending waves.

$$Y_0 = Y_{LOC} + Y_D \qquad (1.3)$$

Y_{LOC} is purely imaginary for a plate of loss-less material, but complex for a real structure. Y_D is purely real for an infinite plate but complex for a finite plate. The real parts determine the power input.

Two quantities are primarily sought in this work, as described in the Summary. The power input P_{IN} can be expressed as the product of the force and the velocity.

$$P_{IN} = \frac{1}{2} \text{Re}\, [F \cdot v_0^*] = \frac{1}{2} F^2 \cdot \text{Re}\, Y_0 \qquad (1.4)$$

or as the sum of the propagating and dissipating powers.

$$P_{IN} = \frac{1}{2} F^2\, \text{Re}\, Y_0 = \frac{1}{2} F^2\, (\text{Re}\, Y_{LOC} + \text{Re}\, Y_D) = P_D + P_{LOC} \qquad (1.5)$$

P_{LOC} (proportional to Re Y_{LOC}) refers to a local dissipation of power in the vicinity of the indenter (the FDI). P_D refers to the power that propagates from the excitation point.The index "D" denotes the direct field around the point. See figure 1.5.

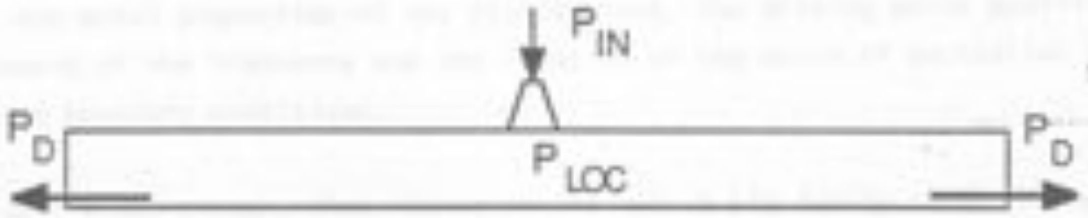


fig 1.5

Figure 1.5. The power flow.

Also the point mobilities and the transfer mobilities are studied. The transfer mobility (Y_{Or}) is the ratio between the total velocity in a point in position r and the force

$$Y_{Or} = \frac{v\ (r)}{F} \qquad (1.6)$$

2. LOCAL ABSORPTION OF POWER IN THE POINT OF EXCITATION

2.1. Approaches to estimate the local absorption of power

Due to the modal properties of the finite plate, the driving point mobility (Y_0) will depend of the frequency and the location of the point of excitation as well as on the boundary conditions.

It is well known though, that the power fed into a big finite plate averaged over several modes, is approximately the same as that fed into an infinite plate that has the same thickness and material properties as the finite plate.

As discussed in chapter 1, in a finite plate Y_0 consists of a real and an imaginary part. For low mobility structures, the local deformation may govern the point mobility, i.e. $Y_{LOC} >> Y_0$.

The FDI-device has been used in all the point mobility measurements. In the measurements of the magnitude of Y_0, the size of the indenter affects the measured value, mainly due to Y_{LOC}. This influence can be predicted.

Consider the relation $\text{Re}\ Y_0 = \text{Re}\ Y_{LOC} + \text{Re}\ Y_0$. If $\text{Re}\ Y_{LOC} << \text{Re}\ Y_0$ the error in the estimate

$$E\ [\text{Re}\ Y_0] = (\text{Re}\ Y_0)_{\text{measured}} \qquad (2.1)$$

is negligible. This error may not be negligible however, which is the task to examine.

This chapter deals with a few different approaches to the problem of estimating the magnitude of the local absorption and to compare it with the measured input power. It should be stressed, that no prediction of this local phenomenon is made in this report but rather an estimate of its importance in the context of power input.

<u>The first way</u> is to measure $\text{Re}\ Y_0$ with increasing size of the indenters, and to study the variation of $\text{Re}\ Y_0$ with increasing frequency. Especially a rapid growth in $\text{Re}\ Y_0$ could indicate a viscous damping. There are other possible explanations to such a growth though. A comparison is made between the theoretical input mobility of an infinite plate (which is an estimate of an averaged $\text{Re}\ Y_0$ for a finite plate) and an expression of the losses derived from a model of the local mobility.

The second way is to compare measured values of the power input at the point and the propagating power, measured according to a statistical model. The propagating power is considered equal to the sum of losses in all modes of the plate (in the steady-state). This requires measurements of the loss factor of the plate and of the spatial averaged mean square velocity of the plate.

A third way of determining a difference between the power inserted at the point and the fraction that really propagates, makes use of a simplifed mobility analogy model, where measurements of the force and two velocities would be sufficient for the calculation. Different difficulties are discussed.

Finally, transfer mobility measurements performed to estimate the "propagating part" (P_D) of the measured P_{IN} is discussed.

2.2. Influence of indenter size

As long as the dimensions of the excitation area is less than approximately a tenth of the flexural wavelength at a certain frequency, and if $\text{Re } Y_{LOC} \ll \text{Re } Y_D$, the size of the Force Distribution Indenter should not affect the real part of the point mobility. Several mechanisms of losses may occur, such as viscous damping and hysteresis, when the indenter causes a local deformation. The deformation is assumed proportional to the force (F) and the radius of the indenter (a). The local deformation of the plate is treated theoretically by a model based on an expression for the average displacement in an elastic half-space under a soft indenter [1]:

$$\langle d \rangle = 0.54 \, \frac{1 - \mu^2}{Ea} \cdot F \qquad (2.2 \text{ a})$$

The magnitude of the point mobility can be divided into real and imaginary parts:

$$|Y_0| = [(\text{Re } Y_{LOC} + \text{Re } Y_D)^2 + (\text{Im } Y_{LOC} + \text{Im } Y_D)^2]^{1/2} \qquad (2.3)$$

If $\text{Im } Y_{LOC} \gg (\text{Im } Y_D, \text{Re } Y_{LOC}, \text{Re } Y_D)$, the magnitude of Y_0 is governed by the imaginary part of the local mobility, i.e.

$$|Y_0| = \text{Im } Y_{LOC} = \omega C \qquad (2.4)$$

The quantity C is the local compliance , which in the case of a static displacement in an elastic half-space by a hard indenter with the radius (a), is

$$C = \frac{1 - \nu^2}{2Ea} \qquad (2.2\ b)$$

Figure 2.1. Local displacement of the plate surface under the FDI.

The expression (2.4) for Im Y_{LOC} is an extension of the static case (eq. 2.2) to higher frequencies in thick plates. C varies with 1/a. An increased indenter radius reduces C, and also Y_0 if ωC is a dominating part of Y_0.

Now, if the local "spring" is not ideal but has losses proportional to C, and if ωC governs Y_0, then an increased indenter radius will reduce both the real and the imaginary parts of Y_{LOC}. If Re $Y_{LOC} > 0$, then both the magnitude and the phase of Y_0 will change differently (when a is altered) than one would expect if Re Y_{LOC} is neglegted.

The discrepancy would increase with the frequency, at least could the differences of the measured value of Re Y_0 be compared to see whether it is an important aspect on the measurement technique. Before making an approach to a numerical study, it should be mentioned again that other mechanisms could give an increasing trend of Re Y_0 with frequency but we assume that an influence from a change of indenter size can be exclusively deduced to local losses. See (4.2)

Preliminary measurements and numerical comparisons

In the preparation of the measurements, preliminary point mobility measurements (Y_m) on the lightly damped concrete slab (object 1 described in 1.1) were evaluated and compared to the real part of a fictive local mobility ($Y_{LOC'}$) calculated with a complex Youngs modulus ($\underline{E} = E\ (1 + j\ \eta_{LOC})$) which gives a real part Re $Y_{LOC'}$. Also the theoretical expression for the driving point mobility of an infinite plate (Y_∞) was included in the comparison. With

$$\underline{C} = \frac{1-\nu^2}{2\underline{E}a} \text{ and } Y_{LOC'} = j\omega\underline{C} \qquad (2.5)$$

one derives

$$Y_{LOC'} = \frac{\omega(1-\nu^2)}{2Ea(1+\eta^2_{LOC})}(\eta_{LOC} + j) \qquad (2.6a)$$

$$\text{Re } Y_{LOC'} = \frac{\omega \eta_{LOC} \cdot (1-\nu^2)}{2Ea\,(1+\eta^2_{LOC})} \qquad (2.6b)$$

The expression for Y_∞ is well known (see e.g. [2])

$$Y_\infty = \frac{1}{8\sqrt{m''B''}} \qquad (2.7)$$

To make a rough comparison between the three quantities, insert some numerical values of the concrete slab.

$\eta_{LOC} = 1.0 \cdot 10^{-2}$ and $10 \cdot 10^{-2}$, $\nu = 0.15$, $E = 2.6 \cdot 10^{10}$ Pa

$2a = 25 \cdot 10^{-3}$ m , $\rho = 2400$ kg/m³ , $h = 0.16$ m

The values inserted in the corresponding expressions give Y_∞ and Re Y_{LOC}. With Y_m from the measurements, the comparison is shown in figure 2.2.

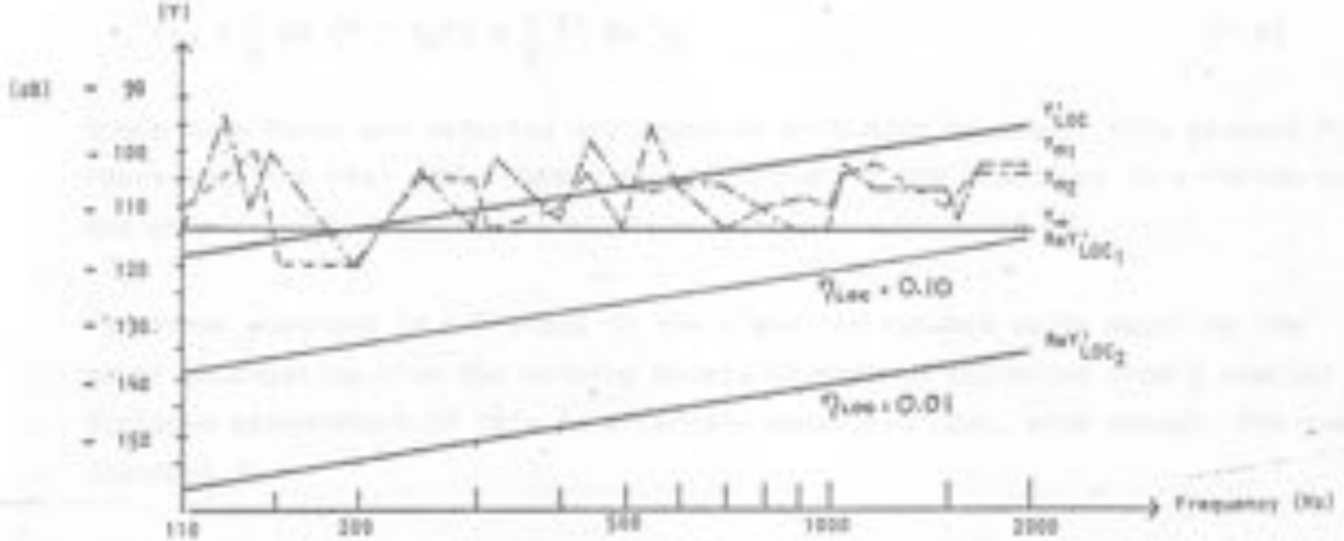


Figure 2.2. Point mobilities.

As expected, since the magnitudes of Y_{LOC}, and Y_m are about equal, the Re Y_{LOC} computed as the fraction η_{LOC} (= 10^{-2}) of ωC is very much below Y_m and Y_∞. If the losses, described with Re Y_{LOC}, should affect the measured value Re Y_m, then the loss factor η_{LOC} must be of the order of 0.1. There are at present no explanations at hand for any mechanisms causing such losses, but concrete is definitely a complicated material to understand and describe. The deformations are of the order of 10^{-9} m if the force is about 1 N:

$$\langle d \rangle = 0.54 \, \frac{1 - \nu^2}{Ea} \cdot F \approx 1.5 \cdot 10^{-9}$$

No loss-mechanisms of any extreme art seem to be likely since no non-linearities were observed.

Note that the indenter should be chosen big enough. Modified impedance heads, as suggested by Adams et al [12] are too small in this context.

2.3. Comparison between driving power supply and power absorbed by the plate.

This approach is based on the assumption that a dissipation of energy locally can be estimated by measuring the power input and the propagating power by means of loss factor and average velocity measurements.

The power input can be calculated from the excitation force (F) and the real part of the driving point mobility as

$$P_{IN} = \frac{1}{2} \text{Re} \, [F \cdot v_0^*] = \frac{1}{2} F^2 \, \text{Re} \, Y_0 \qquad (1.4)$$

Since both force and velocity are measured with high accuracy, this product P_{IN} represents the real power supply as a function of the frequency in a narrow band analysis.

The power absorbed in all modes of the plate, is assumed to be equal to the power propagating from the driving point. It must be estimated from a spatial averaged measurement of $\langle v^2 \rangle$ in a certain bandwidth ($\Delta\omega$), wide enough. The power absorbed is

$$P_{ABS} = \eta \, \omega_c \, m'' \, S \, \langle v^2 \rangle \qquad (2.8)$$

where

- η = loss factor for the whole plate, including dissipation, radiation and losses at the boundaries, in the band $\Delta\omega$.
- ω_c = center "frequency" ($\omega_c = 2\pi$ center frequency) for the band $\Delta\omega$, where $\Delta\omega$ is chosen wide enough to contain at least five eigenfrequencies.
- m'' = mass per unit area of the plate
- S = area of the plate.
- $\langle \hat{v}^2 \rangle$ = spatial averaged mean square velocity

This method to measure and calculate P_{ABS} rises several difficulties.

First, the loss factor η is very difficult to measure with high accuracy [10,11] which is important if a difference between P_{ABS} and P_{IN} shall be significant enough. The loss factor is measured according to

$$\eta = \frac{2.2}{f \cdot T} \tag{2.9}$$

where T is the reverberation time and f is the excitation frequency. T becomes very short, typically 0.2 s at 1 kHz. The plate is suspended on rubber supports, to obtain small losses, since it is easier to measure T if the power losses are low.

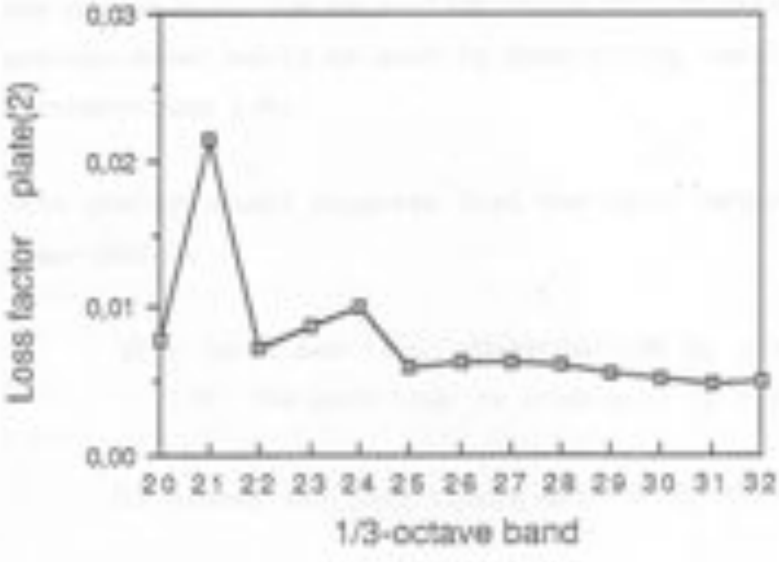


Figure 2.6. Loss factor, measured on plate (2).

Furthermore, also the reverberation time measurement must be both spatial and ensemble averaged. Investigations of the applicability of decay measurements are reported in [10,11]. Modal loss factors through half-power bandwidth measurements is even more difficult to evaluate (the reason being that the local compliance masks the modes on the point mobility plots).

Secondly, a comparison between an averaged quantity within the bandwidth cannot be compared directly with the narrow-band calculated P_{IN} because even if the force is farely constant, the real part of the mobility fluctuates a lot within the bandwidth $\Delta\omega$. The power input occurs basically at the eigenfrequencies for the modes excited which depend on the specific location of the driving point. The bandwidth $\Delta\omega$ must be at least 100 Hz for the typical concrete slab that is considered when $<Q^2>$ is evaluated. The values of P_{IN}, calculated in narrow-band, must be summed to a quantity P_{IN} $(\Delta\omega)$, describing the power input within the bandwidth $\Delta\omega$, then P_{IN} $(\Delta\omega)$ and P_{ABS} will be comparable. The spatial averaging requires very many points. Another way to make the two quantities comparable would be to use inverse filtering of the driving force so that the product $[F^2 \text{ Re } Y_0]$ remains fairly constant within $\Delta\omega$, but this method is not fully applicable at present in our computer system. The conclusion here is that it is very difficult to achieve a significant value of $P_{IN} - P_{ABS}$. See (4.1) for a discussion of the results.

2.4. A mobility analogy model for the plate with local deformation

In the following we study how the velocity at and around the point varies with the distance to the excitation point [8]. Suggest that a simplified mobility analogy model could be used in determining local power absorption (described by a conductance 1/R).

This analogy model supposes that the total velocity of the plate could be described by

a) a local mobility, characterized by the compliance C and the conductance 1/R. The mass that is displaced is neglected.

b) masses and compliances describing the modes of the plate.

c) mass (M) and compliance of the whole plate with its supports.

In 3.2, the deformation caused by a point force nearby the measurement point is discussed further, but for the establishment of the analogy model, a brief consideration is suitable. The deformation of the plate is modelled in chapter 1.

The last kind of motion (c), occurs at frequencies below those of interest, but the second kind (the modes of the plate) are vibrations within the bandwidth considered. For frequencies below $kh = 1$, the simple bending wave equation describes the velocity on an infinite plate at some distance (r) from the excitation point. For higher frequencies, that description clearly underestimates the transversal velocity (v) [9]. The expression for the velocity $v(x,z)$ on a finite plate [[2]]

$$v(x,z) = \frac{4j\omega F(x_0,z_0)}{m'' \cdot b \cdot l} \sum_{n=1}^{\infty} \frac{\phi_n(x,z) \cdot \phi_n(x_0,z_0)}{\omega^2_n (1 + j\eta) - \omega^2} \qquad (2.10\ a)$$

(2.10 a) is not as easily interpreted as the expressions (2.10 b) and (2.11) for the infinite plate, which of course do not take into account the contributions to the velocity from the reverberant field. The deformation at a distance (r) may be described by a term for the propagating waves and a decaying near-field term [2],[9]. The velocity at (r) can be described by

$$v = v_0 \cdot \Pi\,(kr) \qquad \text{for } kr < 4 \qquad (2.10\ b)$$

where $\Pi\,(kr) = H_0^{(2)}(kr) - H_0^{(2)}(-\,jkr)$ and v_0 are determined by the force and the point mobility. If a distance $kr < 0.8$ is chosen, $v_0 > v > 0.8\,v_0$ which gives less than 1 dB change in magnitude within that radius. For distances outside ($kr > 4$), an asymptotic expression is

$$\Pi\,(kr) \approx \sqrt{\frac{2}{kr}}\, e^{-j(kr - \pi/4)} \qquad (2.11)$$

At even greater distances, this expression underestimates the velocity of a finite plate because this direct-field component of the velocity will be superimposed by a diffuse reverberant field, which is constant over the plate when it is averaged in a band $\Delta\omega$. Such a velocity can be described by a statistical calculation [8] (as is detailed in section 2.3).

$$\langle \bar{v}^2 \rangle = P_{IN}/(\eta\ \omega\ m''\ S) \qquad (2.12)$$

Now, the local deformation of the plate remains to be described. The equations for a statical displacement of an indenter in an elastic half-space will approximately describe the local deformation also at higher frequencies [1]. This is in agreement with results presented by Ljunggren [9], which are based on an analysis of local, reactive waves in plates that are thick and/or at higher frequencies ($k \cdot h >> 1$). For a stiff indenter on a half-space, Petersson has worked out the displacement at a distance (r) as

$$d = \frac{(1 - \nu^2) F}{\pi Ea} \arcsin \left(\frac{a}{r}\right) \quad r \geq a \qquad (2.13)$$

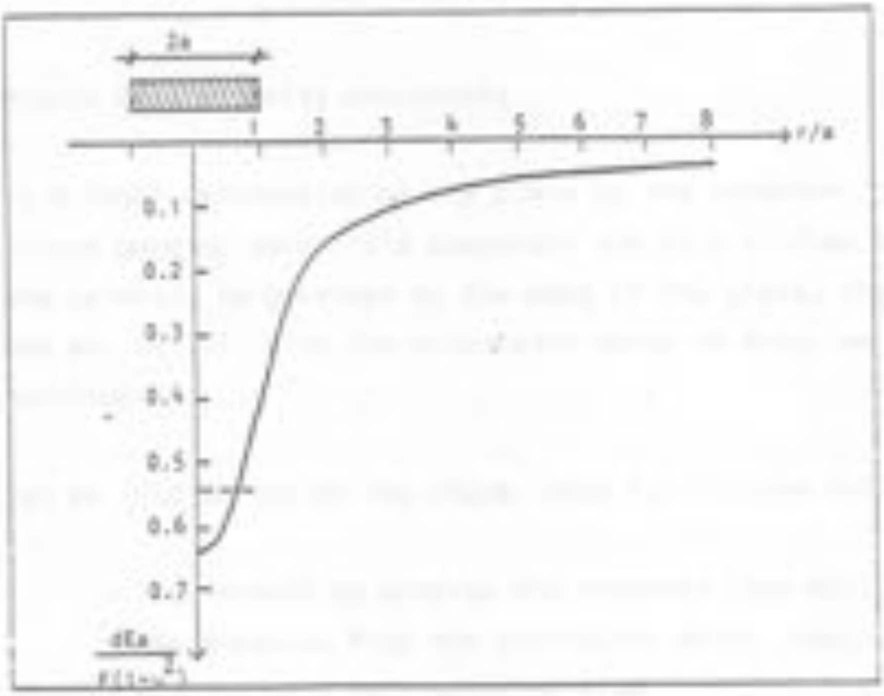


Figure 2.3. Graph of non-dimensional displacement. Rigid indenter on half-space. From [1].

This shows the importance of the location of the transducers in the context of point mobility measurements. The velocity measurements must be straight under the force. If the velocity transducer is applied to the structure only 10 mm apart from the force transducer, then this gives an error in point mobility measurements of 8 dB!

At a distance of, say 8 a, the displacement of the surface of the half-space indicates a reduction of the local deformation of the plate of about 20 dB. That distance is chosen for the measurements. Summarized in the figure 2.4 below, the velocity may be regarded as consisting of three components:

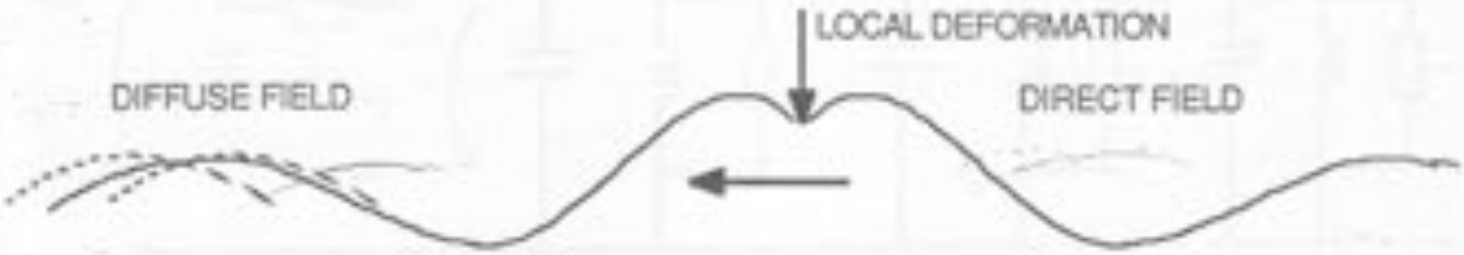


Figure 2.4. Velocity components.

1) A local deformation of the plate by the indenter, 2) a decaying (with kr) direct bending wave field component and 3) a diffuse bending wave field, where the velocity is governed by the mass of the plate, the frequency and the losses. See eq. (2.12). With the discussion above in mind, we are interested in the question:

Can we find points on the plate, that fulfill the following conditions?

- They should be outside the indenter (the FOI).
- The distance from the excitation point should be negligible from a bending wave term point of view.
- The influence of the local deformation must be negligible.
- The direct, bending wave field should dominate over the reverberant field.

This question is discussed more in detail in chapter 3, but suggest for the following analogy model that such points do exist. Measurements presented in figure 3.2 and in chapter 4 and also by Ljunggren [7] indicate that the direct field at the excitation point is 5 - 10 dB above the diffuse-field level. (That is not enough to "separate" the point velocity totally from the modes of the plate.) The following analogy may be an approach to an understanding of what is happening under the indenter when an indenter excites the structure. The analogy model:

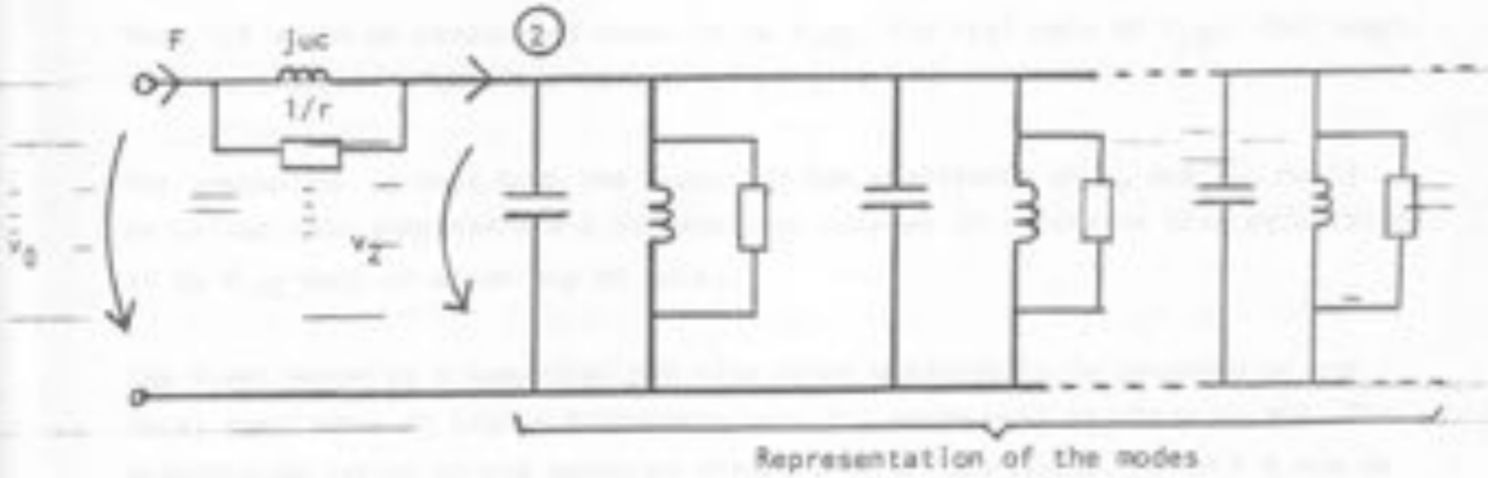


Figure 2.5. An analogy model of the plate.

The idea is to measure v_0, v_2 and F, calculate and/or evaluate the local compliance C, and determine 1/R out from

$$v_0 - \frac{(1/R) \cdot j\omega C}{(1/R) + j\omega C} \cdot F - v_2 = 0 \qquad (2.14\ a)$$

The model is really easy to set up, but is it possible to identify points on the plate where v_2 can be measured? The modal behaviour of the plate influences v_0, which is described by the model. The envelopes of the velocity curves can be employed, the comparison might yield a fairly accurate value of the quantity R. The error in the estimated value C also affects the efficiency of the model.

C and 1/R could be looked upon as a complex local mobility (Y_{LOC}):

$$Y_{LOC} = \frac{(1/R) \cdot j\omega C}{(1/R) + j\omega C} \qquad (2.15\ a)$$

where Re Y_{LOC} represents the losses.

If both the phase and magnitude of v_0 and v_2 could be measured without any influence of the reverberant field, the calculation may yield more precise results: Equation (2.14) reduces to

$$v_0 - Y_{LOC} \cdot F - v_2 = 0 \qquad (2.14\ b)$$

$$Y_{LOC} = \frac{1}{F}(v_0 - v_2) = Y_0 - Y_{20} \qquad (2.15\ b)$$

$$\text{Re } Y_{LOC} = \text{Re } Y_0 - \text{Re } Y_{20}$$

Then 1/R would be considered equal to Re Y_{LOC}, the real part of Y_{LOC}. The imaginary part ought to be close to that of ωC [1].

The assumption is that both the phase and the magnitudes of Y_0 and Y_{20} could be relied upon, and there are at least two sources of errors in this calculation of Re Y_{LOC} each of which may be fatal.

The first error is a numerical one. The point mobility Y_0 is governed by the local compliance at higher frequencies and the phase will be close to 90°. The standard deviation of the measured phase is hard to suppress below 1 % due to the signal analysis procedure employed. Other errors may arise that affects the phase even more. Re Y_0 is therefore much smaller than the magnitude of Y_0. Also the real part of the transfer mobility is a small quantity. The subtraction may therefore yield results that are almost random.

The second kind of error is very difficult to overcome. In analogue circuits with lumped elements, the phase of a transfer function, is assumed to be caused by the components, and the phase shift caused by the travel time delay is neglected. In the structural-acoustic case, this assumption is not applicable. Even at small distances, the timedelay is not negligible. This could have been compensated for if the wave-number k could be described with the bending-wave equation only. That compensation is a factor, e^{-jkr}. The velocity at a point at the distance r is caused by several kinds of waves and the time-delay phase shift cannot be compensated for that simple. Measurements have confirmed that the phase differs approximately with a factor e^{-jkr}, but since the calculation of Re Y_{LOC} requires very accurate phase information, it seems as if the analogy model with lumped elements must be limited in this case to a comparison of magnitudes only. On the other hand, this requires a reliable value of C which is not easily achieved. See (4.1) and (4.4a).

There is however another approach that doesn't give these difficulties. If a transfer mobility is measured using an accelerometer on the opposite side of the plate, straight under the exciter and FDI, then the local compliance ought to be "short-cut". The real part of that transfer mobility is equal to the real part of the mobility Y_0. In the initial part of the project, we did not make such an assumption because we could not show that the two mobilities were equal. It was made clear very recently by Ljunggren [9], that the real part of the transfer mobility is equal to the real part of the point mobility, which makes such measurements interpretable in this context. By comparing these mobilities, an eventual discrepancy would indicate very strongly that there really occurs a local absorption of power (which is the issue here). The evaluation of the measurements is in (4.4b).

2.5. Transfer mobilities as an estimate of the real part of the point mobility

In an ISVR-paper [6] a very promising method to measure the real part of the point mobility is developed, where only the propagating power is measured, i.e. Re Y_D is estimated. It is shown that if the losses of the plate are small, and a reverberant field is excited in the plate at a position (A), the transfer mobilities between A and two remote points in the reverberant field (B,C) yields an estimate of Re Y_D as

$$E\,[\mathrm{Re}\,Y_D] = |Y_{AB}| \cdot |Y_{AC}| / \hat{Y}_{BC} \qquad (2.16)$$

The transfer function $\hat{Y}_{BC}$ means an envelope on the peaks in Y_{BC}. If another pair of points (DE,FG.....) are measured additionally, the maximum of E [Re Y_D]] at every frequency should be used. See section (4.3).

(Note that this method is applicable in cages where a real part of a point mobility must be measured but no force measurements can be made, for instance under a machine footing.)